# **Hydrogen-like atom**

*Consider*[*hydrogen-like atom in the ground state (1s)*](https://www.dsedu.org/courses/dft/h-atom)*. Write analytical expression for lowest eigenvalue and corresponding eigenfunction for hydrogen-like atom. Make programming module with this solution.*

*Tips: use Schrödinger equation in spherical coordinates and use symmetry condition for lowest energy state.*

*Literature: Pauling, L.; Wilson, E.B. Introduction to Quantum Mechanics, McGraw-Hill, New York, 1935*

Hydrogen like atom has nucleus and one electron. C5+ ion, for example, has one electron and nucleus with charge *Z* = +6.

Electron is moving in central potential of nucleus



where *r* is the distance from the nucleus.

Please note, that all formulas we are writing in atomic units (a.u.) where



For spherical symmetry we can look for the solution of the Schrödinger equation in the form



where  is spherical harmonics. Moreover, the Schrödinger equation for spherical symmetry can be separated on two independent equations for *r* and .

The Schrodinger equation in spherical coordinates for part depending on *r* only, the radial Schrodinger equation is



The lowest energy state of H-like atom is 1*s* state with *n* = 1, *l*= 0, where *n* is the principal quantum number, *l* is the angular quantum number. This 1*s* state has spherical symmetry and wave function of this state does not depend on spherical angles θ and ϕ:



For 1*s* state (*n* = 1, *l* = 0) the exact analytical solution [[[1]](#endnote-1)] of Eq. is



In order to edit formulas you need to install **Math Type Editor** <https://www.wiris.com/en/mathtype/>

Automatic references and formula numbering feature are used.

1. Pauling, L.; Wilson, E.B. Introduction to Quantum Mechanics, McGraw-Hill, New York, 1935. [↑](#endnote-ref-1)